

Nonparametric Predictive Inference for option pricing based on the binomial tree model

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Abstract

Nonparametric Predictive Inference (NPI) is a frequentist statistical method based on only few assumptions, which has been developed for and applied to, several areas in statistics, reliability and finance. In this thesis, we introduce NPI for option pricing in discrete time models. NPI option pricing is applied to vanilla options and to some types of exotic options.

We first set up the NPI method for the European option pricing based on the binomial tree model. Rather than using the risk-neutral probability, we apply NPI to get the imprecise probabilities of underlying asset price movements, reflecting more uncertainty than the classic models with the constant probability while learning from data. As we assign imprecise probabilities to the option pricing procedure, surely, we get an interval expected option price with the upper and lower expected option prices as the boundaries, and we named the boundaries the minimum selling price and the maximum buying price. We also prove that the put call parity property of the classic model is also followed by the NPI boundary option prices. To study its performance, we price the same European options utilizing both the NPI method and the Cox, Ross, and Rubinstein binomial tree model (CRR) and compare the results in two different scenarios, first where the CRR assumptions are right, and second where the CRR model assumptions deviate from the real market. It turns out that our NPI method, as expected, cannot perform better than the CRR in the first scenario with small size historical data, but as we enlarge the history data size, the NPI method's performance gets better. For the second scenario, the NPI method performs better than the CRR model.

We also present the American option pricing procedure from an imprecise statistical aspect. We propose a novel method based on the binomial tree. We prove through this method that it may be optimal for an American call option without dividends to be exercised early, and some influences of the stopping time toward option price prediction are investigated in some simulation examples. The conditions of the early exercise for both American call and put options are derived. We also study the performance of the NPI pricing method for American options via simulation in the same two scenarios as the European options. Through the performance study, we conclude that the investor using the NPI method behaves more wisely in the second scenario than the investor using the CRR model, and faces to more profit and less loss than what it does in the first scenario.

The NPI method can be applied to exotic options if the option payoffs are a monotone function of the number of upward movements in the binomial tree, like the digital option and the barrier option discussed in this thesis. Otherwise, either we can manipulate the binomial tree in order to assign the upper and lower probabilities straightforwardly, for instance the look-back option with the float strike price, or a new probability mass is needed to be assigned to the payoff binomial tree according to the option definition which is attractive and challenging for future study.