# Probability & Statistics III (Term 2) - Tutorial 4

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**Problem 1.**

(a) For $X \sim N(0,1)$, find:

1. $P(X < 0)$
2. $P(X \leq 0)$
3. $P(X < 1.63)$
4. $P(X > 0.57)$
5. $P(X < -8.32)$
6. $P(X > -1.96)$
7. $P(X < 1.96)$
8. $P(-1.96 < X < 1.96)$
9. $P(-2.10 < X < 0.50)$
10. $u$ such that $P(X < u) = 0.95$
11. $v$ such that $P(-v < X < v) = 0.90$
12. $w$ such that $P(X > w) = 0.99$

(b) For $Y \sim N(128, 4)$, find:

1. $P(Y > 128)$
2. $P(Y > 215)$
3. $P(Y < 130)$
4. $P(Y < 124)$
5. $P(124 < Y < 132)$
6. $P(125 < Y < 134)$
7. $s$ such that $P(Y > s) = 0.95$
8. $t$ such that $P(128 - t < Y < 128 + t) = 0.99$

**Problem 2.**

A method for weighing extremely light objects gives results (in micrograms) for nine weighings of a particular specimen with sample mean of 124. Suppose the specimen's actually weight is $\mu$ micrograms, that the known accuracy of the measurement method can be described by the model $N(\mu, 10)$ for measurements of an actual weight $\mu$, and that your prior knowledge about $\mu$ is taken into account via prior distribution $\mu \sim N(125, 5)$.

(a) Derive the posterior distribution for $\mu$, and the corresponding posterior predictive distribution for a future measurement $X_{10}$ of the same specimen, using the same measurement method.

(b) Calculate the posterior probability for the event $\mu > 125$.

(c) Calculate the posterior probability for the event $\mu < 123$.

(d) Find $u$ such that the posterior probability for the event $122 < \mu < u$ is 0.90.

(e) Find the interval of minimal length which contains $\mu$ with probability 0.90, according to the posterior distribution.

(f) Calculate the posterior predictive probability for the event $X_{10} > 125$.

(g) Find the interval of minimal length which contains $X_{10}$ with probability 0.95, according to the posterior predictive distribution.

(h) Suppose we wish to determine $\mu$ very accurately by taking further measurements of the same specimen, using the same measuring method. Assume that one wants to achieve an interval which contains $\mu$ with posterior probability 0.95, and with a length of at most 1. How many further measurements must we made?