Probability & Statistics III (Term 2) - Tutorial 3

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Problem 1.
Elvis Presley had a twin brother (who died at birth). What is the probability that Elvis was an identical twin? Assume known: approximately 1/125 of all births are fraternal twins and 1/300 are identical twins, and approximate the probability of a boy or girl birth as 1/2.

Problem 2.
A box containing eight parts is received from a supplier. In the past, 70% of all such boxes have had zero defective parts; 20% have had one defective part, and 10% have had two defective parts. We assume therefore that all boxes containing eight parts will have either 0, 1 or 2 defectives, in the proportion as given. Three parts are selected at random from the box of eight, and one part is found to be defective.

(a) What is the probability that the box of eight parts received from the supplier actually contains two defective parts?

(b) Suppose you take one more part, at random, out of the box, and this is not defective; answer part (a) again with this extra information.

(c) What is the probability that, after this information on the first four randomly selected parts from this box, a fifth part selected from it, again at random, is defective?

Problem 3.

(i) Suppose that birthdays are in any month of the year with equal probability 1/12. You are at a party, where you meet John, Paul and Simon, who you never met before (no twins involved), and you hear that John's birthday is in a later month of the year than Paul's birthday, but you do not actually hear which months. What is the probability that Simon's birthday is in an earlier month of the year than Paul's birthday?

(ii) (*An extra challenge for those who are very keen!*)
What would happen if, for the similar setting as in part (i) with similar assumptions, the unit 'months' was replaced such that the year was divided into n periods of equal length (say, e.g. $n = 52$ for weeks, $n = 365$ for days, *et cetera*). Derive the answer to the question in part (i) for general n, and comment on the answer for the limiting case $n \to \infty$.
Note: You may wish to use that:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad \text{and} \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$