Probability & Statistics III (Term 2) - Tutorial 2

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Problem 1.
An investor with assets of £10,000 has an opportunity to invest £5,000 in a venture that is equally likely to pay either £15,000 or nothing. The investor's preferences can be quantified by utility function $U(£x) = \ln x$, where x is his total wealth.

(a) What should the investor do? Investigate the sensitivity of his investment decision with regard to possible changes in his current assets.

(b) Discuss the effect on the investor's decision if his utility function would instead have been $U_1(£x) = \ln\left(\frac{x}{1{,}000}\right)$.

(c) Suppose that the investor has the opportunity to consider different amounts for the investment, so he can invest £m, where $0 \leq m \leq 10,000$, where the outcome of the venture is equally likely to be either £$3m$ or nothing. For which values of m would he choose to invest? Discuss this, also in relation to the sensitivity analysis performed in part (a).

(d) Suppose the investor can place a bet with a friend before making the investment decision. The bet is as follows: if a fair coin lands heads up, the investor wins £1,000 from his friend, but if it lands tails up, the investor pays £1,000 to his friend. Only after the bet has been resolved will the investor decide whether or not to invest £5,000 in the venture. Represent the investor's decision problem via a decision tree. Describe the optimal strategy of the investor, i.e. should he place the bet with his friend, and if so (or if not), should he invest in the venture? Comment briefly on this optimal strategy.

Problem 2.
A person with utility function $U(£x) = \ln x$, where x is his total wealth, has a choice between the following two alternatives: A - win £10,000 with probability 0.2; win £1,000 with probability 0.8; and B - win £3,000 with probability 0.9; lose £2,000 with probability 0.1. If he currently has £2,500, which alternative should he choose? Also derive what he should do if he currently has £5,000, and what if he currently has £10,000. Comment on these choices between A and B.