Probability & Statistics III (Term 2) - Tutorial 1

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Problem 1

Solve the following decision problems by maximising Expected Money Value, and find the expected value of perfect information in each case. (The rewards in the payoff tables are in pounds).

(a)

	θ_1	θ_2
d_1	130	90
d_2	100	105
Prob	1/3	2/3

(b)

	θ_1	θ_2
d_1	1000	100
d_2	100	200
Prob	0.2	0.8

Problem 2

A firm is considering launching one of two products, d_1 or d_2. The former is expected to do well if the economy is good, the latter may do better if the economy is weak. The estimated returns (in £1,000) and the relevant probabilities are:

	θ_1: good	θ_2: moderate	θ_3: weak
d_1	50	30	20
d_2	40	30	30
Prob	0.3	0.4	0.3

A market research firm offers to provide, for £4,000, a survey of the prospects. Should the offer be accepted, assuming the firm aims at maximum expected money value?

Suppose that the managers agree strongly that the probability of a good economy is 0.3, but have major doubts about the probabilities of moderate and of weak economy. Perform a sensitivity analysis relevant for this situation, and discuss the result with regard to the market research option.

Problem 3

On his 20th birthday a patient is brought into hospital with an illness which is either Type I, with probability 0.4, or Type II, with probability 0.6. Independent of the type of illness, without treatment he will die on that day with probability 0.8 and otherwise survive and have normal life expectancy.

The surgeon has three possible courses of action open to her:

(1) not to treat the patient;

(2) to give the patient a drug once;

(3) to operate on the patient once.

She cannot both operate and administer the drug. Both operating and administering the drug are dangerous to the patient. Independently of the type of illness, operating on the patient will kill him with probability 0.5 and the drug will kill him with probability 0.2.

If the patient survives the poisonous effects of the drug, it will either cure him or have no effect, each with probability 0.5, if he has Type I illness; and will have no effect if he has Type II illness. If the patient survives an operation, it will cure him with probability 0.8 if he has Type I illness and with probability 0.4 if he has Type II illness, otherwise having no effect. Survival of the patient will give him a life expectancy of 70 years in all cases.

(a) Draw a decision tree to represent this problem. Determine the surgeon's best strategy assuming she wishes to maximise her patient's residual life expectancy.

(b) Calculate the expected value of perfect information, in terms of life expectancy, and comment briefly on this value.

(c) Suppose that there is reason to doubt the chosen value (0.8) of the probability of immediate death in case of no treatment. Perform a detailed study of the optimal strategy as function of this probability, and comment briefly on the sensitivity of the surgeon's optimal strategy with regard to the value $p = 0.8$.