Probability & Statistics III (Term 2) - Homework 6

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Problem 1.
The proportion of people who currently have a rare blood disorder is 0.001. A test for it returns one of two results: positive, which is some indication that you may have the disorder, or negative. The test may give the wrong result: if you have the disorder, it will give a negative reading with probability 0.05: likewise, a false positive result will happen with probability 0.05.

(a) You have three independent blood tests and they are all positive. Do you have a high probability of having the disorder?

(b) Given the three positive results, what is the probability that a fourth independent blood test will also give a positive result?

Problem 2.
In a study on the effects of diet on infant haemoglobin levels a researcher arranged for one randomly selected member of each of 6 sets of identical twins to drink milk with honey each night, while the other got milk without honey. For each pair the haemoglobin levels of both twins were measured at the end of a test period. The researcher attached probability $1/2$ to the hypothesis that the honey has no effect and $1/4$ each to the hypotheses that the twin receiving honey will have a higher haemoglobin level with chance $3/4$ and 1, respectively.

(a) In the study all six of the twins receiving honey had the higher haemoglobin level. Calculate the researcher's posterior probabilities for the three hypotheses considered.

(b) Suppose that a seventh pair of identical twins is found who are willing to join the study. What is the researcher's predictive probability that the twin given honey will have the higher haemoglobin level at the end of the test period?

Problem 3.
An unseen coin is known to be either fair or double-headed. When tossed, a fair coin will land heads or tails, each with probability $1/2$. Different tosses with the same coin are independent. Suppose that, prior to getting observations from tossing the coin, you think it equally likely that the coin is fair or double-headed. Suppose the coin is tossed $n \geq 1$ times. Derive the posterior probability for the event that the coin is fair, for all possible combinations of n observations. Also derive the corresponding posterior predictive probabilities that toss $n + 1$ gives heads, and briefly comment on these results.