Probability & Statistics III (Term 2) - Homework 1 & 2

Frank Coolen

The problem:
The manager of a company must decide whether to manufacture a new product, and if so, whether to build a large or small production plant. The planning period is 10 years. The crucial unknown factor is the size of the market for the product. A big plant, if built now, must be run for the next ten years. A small plant, if built now, could be expanded after two years. (More delayed building schemes are not considered, as they would allow competitors to dominate the market, if demand is high).

Marketing information:
The marketing section suggest that there is a:
60% chance that demand will be high for the whole ten year period,
30% chance that demand will be low for the whole ten year period,
10% chance that demand will be high for the first two years and then fall to low for the next eight years.

Accounting information:
The accounting section put forward the following estimates:
A large plant with high demand would yield 2 million (dollars) annually;
A large plant with low demand would yield 0.2m annually (due to high fixed costs);
A small plant with low demand would yield 0.8m annually (being more economical to run);
A small plant with high demand would yield 0.9m annually for the first two years, falling to 0.5m annually thereafter (due to competition from other producers);
If an initially small plant were expanded after two years and demand stayed high, then it would yield 1.4m annually for the remaining eight years;
If an initially small plant were expanded after two years and demand fell from high to low, then the plant would yield 0.1m annually for the remaining eigth years.

Capital costs:
Estimates from the construction section suggest that:
A large plant would cost 6m;
A small plant would cost 2.6m;
The expansion costs from small to large after two years would be 4.4m.

Homework 1 Exercise:
Should the company build? If so, should the initial plant be large or small? Give advice on the optimal actions to take over the entire period (so now, and possibly after two years), aiming at maximum expected money return over the ten year period. Start by drawing the decision tree, and calculating all relevant pay-offs and probabilities.

Homework 2 Exercise:
(a) Find and interpret the expected value of perfect information.
(b) Discuss whether expected money return is a sensible way of choosing a decision in this problem.
(c) Carry out a sensitivity analysis on the effect on the decision procedure of varying the probabilities of high sales for the whole period and of high sales for the first two years with low sales thereafter, keeping the probability of low sales throughout fixed at 30%.