Decision Theory III - Solution Homework 8 (April 2015)

The solutions presented below are just short solutions with some additional comments: the mathematics to derive these is pretty straightforward, the decision trees are easy to draw.

Problem 1

(a) The optimal strategy is to keep choosing a new card until one either the STOP card appears or 100 (in units of £1,000), whichever of these two occurs first. It is easy to see that each of these will occur first with probability 1/2, so the expected pay-off is 50, which indeed also follows from the detailed analysis using a decision tree.

(b) Replacing the pay-offs by the corresponding utility values, in the decision tree, leads to optimal strategy to stop play after the first card, in all cases. This has expected utility 7.758.

(c) The risk profiles, with payoffs in pounds, are $P(0) = P(100,000) = \frac{1}{2}$ for part (a), and $P(0) = P(15,000) = P(20,000) = P(100,000) = \frac{1}{4}$ for part (b). In part (a), the player is risk neutral and the possible large gain of 100,000 ensures that the game continues if possible until that amount comes up. In part (b), the player is risk averse and does not wish to risk losing any positive sum that may come up as first card.

Problem 2

(a) $P(S_1|A_1) = 0.8$ and $P(S_1|A_2) = 0.2$, using Bayes’ theorem.

(b) It is optimal to take the market research and follow its advice. This leads to expected pay-off 30 (all in units of £1,000).

(c) The risk profile of the optimal solution is $P(90) = 0.4, P(0) = 0.5, P(-60) = 0.1$. She may not be able to cover the possible loss of 60,000, so she may want to use a utility function to reflect severity of this outcome more (aiming at an optimal solution which avoids this possible loss).

(d) (i) The decision tree will show, by considering the difference in expected pay-off between the branches related to going for market research or not, that its expected value is 15,000. (ii) With perfect information about advice and sales, the best possible outcome is 90 with probability 0.5 and also 0 with probability 0.5, so the expected pay-off is 45. The expected value of perfect information is therefore 45-30=15. So, compared to the original problem, by improving its reliability the market research could be worth up to 15,000 more, hence in total it could have value 30,000.