These are two old exam questions related to material from Term 1, which are set to assist with revision. The first was a Section A question (with some additional bookwork), the second Section B.

**Problem 1** (Exam 2013 Q.1)

A game consists of four cards, one with the word STOP on it and three with amounts of money: 15,000, 20,000 and 100,000 GBP, respectively. A player chooses a card: if it is the STOP card then the game is over and the player wins no money. If it reveals an amount of money, the player can either decide to stop, winning that amount, or continue the game by selecting another card. At any time, the player may decide to quit and keep the largest amount of money revealed on any chosen card up to that moment, or to continue and get a further card. However, if the STOP card is chosen, the game ends and the player wins no money. At any stage, each card that has not yet been chosen before has equal probability to be selected.

(a) Draw a decision tree representing all decisions the player may have to make in this game. Solve this game, assuming that the player aims to maximise expected money value.

(b) Suppose that the player’s utility for winning \( X \) GBP is \( u(X) = \ln(1 + X) \). Solve this game, assuming the player aims to maximise utility.

(c) Compare the results to parts (a) and (b); include in your discussion consideration of the risk profiles of the two respective optimal decision sequences and the attitude to risk as reflected by the utility function \( u(X) \) in part (b).

**Problem 2** (Exam 2011 Q.7)

Christine, a beer trader, is considering whether to market a particular beer (decision \( d_1 \)) or not (decision \( d_2 \)). If decision \( d_1 \) is taken, then she has probability 0.5 that sales will be good (event \( S_1 \)) and probability 0.5 that sales will be bad (event \( S_2 \)). Given good sales, overall expected profits are £90,000, while given bad sales expected losses are £60,000. If decision \( d_2 \) is taken, then there will be no profit or loss. Before deciding whether to market the product, she can commission some market research (\( m_1 \)), or not (\( m_2 \)). The research will either reach a positive recommendation (event \( A_1 \)) or a negative recommendation (event \( A_2 \)). The research is judged to be 80% reliable, i.e.

\[
P(A_1|S_1) = P(A_2|S_2) = 0.8.
\]

(a) Find the conditional probability of good sales, conditional on the market research reaching a positive recommendation. Similarly, find the conditional probability of good sales, conditional on the market research reaching a negative recommendation.

(b) Draw and solve the decision tree for the above problem, assuming that the market research is free and that Christine wishes to maximize expected money value. Present the optimal solution.

(c) Find the risk profile of the optimal solution obtained in part (b). Discuss what further considerations would be relevant to help Christine decide whether the best decision has been chosen.

(d) Find and interpret:

(i) The expected value of information of market research (i.e. how much is the market research worth?).

(ii) The expected value of perfect information about advice and sales, assuming that the market research is free.