The solutions presented below are just short solutions with some additional comments: the mathematics to derive these is pretty straightforward, using the graphical method to solve two-person zero-sum games according to John Nash’ minimax theory. Of course, for full solution all steps must be included, notation must be carefully introduced, and all optimal strategies for both players must be identified and summarized, together with the value of the game.

**Problem 1**

C3 is dominated by both C1 and C2, hence R will never play it. The graphical method indicates that the three line segments representing C1, C2 and C4 all cross at $p^* = 1/2$ (it is needed to show this mathematically; trivial but important as the plot may not be fully accurate), so the minimax strategy for R is $P(R1) = P(R2) = 1/2$. The corresponding value of the game is $V = 3$. The main challenge of this question is to find all minimax strategies for C.

First, C can restrict to playing either C1 or C4. By symmetry (or just by doing the calculations as shown in the lectures), the minimax strategy is $P(C1) = P(C4) = 1/2$.

Secondly, C can restrict to playing either C2 or C4. Applying the method to solve $2 \times 2$ games straightforwardly lead to minimax strategy $P(C2) = 2/3$ and $P(C4) = 1/3$.

(Note that C cannot restrict to playing either C1 or C2, as then R2 would dominate R1 for R, hence C would play C2 with resulting pay-off 4: this is not a minimax strategy.)

Finally, and actually providing all solutions simultaneously, C can play any combination of the three strategies C1, C2 and C4, which leads to value 3, and where $P(C4) > 0$ (see previous note). The analysis, although not directly shown in the lectures, is straightforward: Let $q_i = P(Ci)$, so the probability with which C plays $Ci$, for $i = 1, 2, 4$. For C to ensure that $R$ cannot benefit from possible superior skills, the expected pay-offs to $R$ for strategies $R1$ and $R2$ must be equal, hence $q_1 + 2q_2 + 5q_4 = 5q_1 + 4q_2 + q_4 = 3$, where the value follows from the minimax theorem. Together with $q_1 + q_2 + q_3 = 1$ and the requirement $0 \leq q_i \leq 1$ for $i = 1, 2, 4$, this leads to solutions $(q_1^{*}, q_2^{*}, q_4^{*}) = (3x - 1, 2 - 4x, x)$ with $1/3 \leq x \leq 1/2$. Clearly, the two solutions above are the boundary cases and $q_4^{*} > 0$.

**Problem 2**

This problem is close to Problem 1, again C3 is dominated by C1. For $a = 2$, the solution is straightforwardly derived: $P(R1) = 2/5$, $P(R2) = 3/5$, $V = 13/5$, $P(C2) = 4/5$ and $P(C4) = 1/5$.

For the sensitivity analysis, the game must be solved for all values $1 \leq a \leq 4$ (to my surprise many students only considered the integer values!). For $1 < a < 3$, the solution again corresponds to the point where C2 and C4 cross: $P(R1) = \frac{2}{7-a}$, $P(R2) = \frac{5-a}{7-a}$, $V = \frac{15-a}{7-a}$, $P(C2) = \frac{4}{7-a}$ and $P(C4) = \frac{3-a}{7-a}$.

For $3 < a \leq 4$ the solution does not depend on $a$ and corresponds to the point where C1 and C4 cross, it follows directly by symmetry: $P(R1) = P(R2) = 1/2$, $V = 3$, $P(C1) = P(C4) = 1/2$.

Finally, and most interestingly, consider $a = 3$. Now C can play any combination of C1, C2, C4, which all cross at $p = 1/2$, so $P(R1) = P(R2) = 1/2$ and $V = 3$. With probabilities $q_i = P(Ci)$, similar reasoning as in Problem 1 leads to equations $q_1 + 3q_2 + 5q_4 = 5q_1 + 3q_2 + q_4 = 3$ and $q_1 + q_2 + q_4 = 1$, which leads to solutions $(q_1^{*}, q_2^{*}, q_4^{*}) = (\frac{1-a}{2}, x, \frac{1-a}{2})$ for all $0 \leq x \leq 1$. Note that $q_2^{*} = 0$ gives the same solution as we had in the range $3 < a \leq 4$, and that $q_2^{*} = 1$ is also possible here, indeed it simply leads to guaranteed pay-off 3.

It is easy to check that the solution to this problem, originally with $a = 2$, is not very sensitive to small changes in this value: within the substantial range $1 \leq a \leq 4$ (so from half to double the first suggested value), the optimal strategy for R varies from $P(R2) = 1/2$ to $2/3$ and the value of the game varies from $V = 7/3$ to 3.