Decision Theory III - Term 2, Homework 1 (February 2015)

Problem 1
This can be considered to fit in the setting of Arrow’s theorem: each exam provides a ranking, which can be considered as an individual preference ordering over the 40 students. Hence, the set of 6 preference orderings resulting from the 6 exams can be considered to be a preference profile. The task of the Board of Examiners is to combine these 6 orderings to an overall ranking, hence a preference ordering. Arrow’s theorem applies therefore, hence there does not exist a method to combine such exam results that ensures that Arrow’s axioms are satisfied (these can also be translated to this specific setting, it is not required and left as an exercise). Note that it is assumed here, in line with Arrow’s theory, that only orderings, so rankings, are used, not actually exam marks. With the latter, under assumptions of ‘anonymity’ and ‘strong pareto’, and scaling marks appropriately (this is left as something to think about!), one could use Harsanyi’s ‘utilitarianism’ theory and get to an overall ranking by summing each student’s individual exam marks. However, it is often argued (at Board of Examiners meetings) that comparability of marks for different modules is very difficult, so the use of rankings only may not be unrealistic.

Problem 2
(a) In round 1 $B$ has the fewest votes (0) and is deleted. In round 2 $D$ is deleted (1 vote). Voter 5 will next vote for $A$, hence in the final round $C$ is deleted as 3 people vote for $A$ and 2 for $C$. So the group preference order is $B <_g D <_g C <_g A$.

Let us first consider axiom (U): the resulting group preference is easily seen to be a full ordering which is transitive, so satisfies (O1) and (O2). However, individuals are not allowed to include indifferences in their preference order, which is of course a limitation on the preference profiles to which the method applies, hence (U) does not hold. Axiom (D) clearly holds because everybody’s preference order is taken into account and can change the group preference. Axiom (P) also holds because if all voters prefer location $X$ over location $Y$ then $Y$ will certainly be deleted from the voting procedure in an earlier round than $X$, hence the group will prefer $X$ over $Y$. Note that here the vote-off plays a role: it is possible for all voters to prefer $X$ over $Y$, yet neither of these getting any votes; in this case, the vote-off guarantees that $Y$ would actually be deleted from the process before $X$, hence the group preference order would also have $X$ preferred over $Y$.

This example illustrates that axiom (I) does not hold, because were it to hold then the group preference between $A$ and $B$ would not depend on the presence or absence of $C$ and $D$. However, with $C$ and $D$ included we have $B <_g A$, but if $C$ and $D$ had not been included, then two individuals held preference $B <_i A$ while three held preference $A <_i B$ hence in the single round of voting $A$ would be deleted leading to $A <_g B$.

(b) $B$ may feel that this voting procedure was particularly unkind to them, because not only was it everybody’s second favorite location, it also beat each of the other candidates in a direct pairwise vote if the simple majority rule had been used.