DECISION THEORY EXERCISES:
GAME THEORY

1. The Gambler

(a) A gambler suggests that you play the following game. You each toss a coin. If you get heads (H) and he gets tails (T), you win £30. If you get T and he gets H, you win £10. If the coins match, you lose £20. You find that you have no coins. “Never mind”, says the gambler, “we can still play the game, instead of tossing coins, each time I’ll write down H or T and you do the same. We score as before.” Should you play? (i.e. find the value of the game.)

(b) Suppose that you decline to play the game. The gambler offers you the following alternative. “You write down the outcome of one toss, I’ll write down the outcome of two tosses. Payoffs will be as follows”:

Him: HH HT TH TT
You: H 1 0 -1 -2
T -2 -1 0 2

(For example, if you play H and he plays TH then you pay him a pound.)

Analyse this game using a graphical method (i.e. find the value, strategies for both players, whether you should play, by each method).

2. Minimax Solution: Graphical Solution

Suppose that in a particular zero sum game the payoffs to R are as follows:

<table>
<thead>
<tr>
<th>R:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Use a graphical method to solve the game (include the strategies for both players).

3. Another 2 x n Game

Solve the two-person zero-sum game, with payoffs given below (with payoffs as shown to R, and the negative of those payoffs to C), by use of a geometric method. Describe which strategy each of the players should choose corresponding to the solution.

<table>
<thead>
<tr>
<th>C:</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>0</td>
<td>-3</td>
<td>6</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>R2</td>
<td>1</td>
<td>4</td>
<td>-2</td>
<td>-1</td>
<td>-4</td>
</tr>
</tbody>
</table>

4. Two Burglars (*)

(This problem explores what happens in the repeated prisoners’ dilemma when the number of games is random. This is not directly an exam-style question, as we have only paid little attention to such games in the lectures. If such a question were to be asked in the exam, more guidance would be given instead of just asking you to analyse the game.)

Burglars Bill and Betty are arrested. The police have enough evidence to lock them up for a Small Job. They know, but cannot prove, that they also committed a Big Job. Bill and Betty are separated and each is offered the following deal. If neither confesses, then each will serve u years for the small crime. If one confesses, and the other does not, then the burglar who confesses will serve v years, the other w years. If both confess, they will each do x years. The values are such that v < u < x < w.
Suppose further that the police have evidence to convict Bill and Betty of several such jobs. For each job, there is a small version for which they can convict both burglars, and a big version which they can only convict on if at least one burglar confesses. Each crime is handled in sequence with ‘scoring’ as above. Bill and Betty are not allowed to communicate, but they are told, after each crime is treated, whether or not their partner confessed.

Suppose also that Bill and Betty do not know how many crimes they will be accused of. Suppose in particular that after each crime has been settled the police toss a coin which has probability $p$ of landing heads. If the coin lands heads, then Bill and Betty are accused of the next crime, while if the coin falls tails the process stops. [Not realistic—but it gives us a stopping rule which is easy to analyse.]

Analyse the game. In particular, show that for any $v < u < x < w$ there are certain values of $p$ for which never confessing is an equilibrium strategy for Bill and Betty (i.e. devise a strategy such that if either burglar follows the strategy then both burglars can do no better in expected payoff than never confessing).