Scattering Amplitudes, MHV Diagrams, and Wilson Loops

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Brandhuber, Spence, GT

hep-th/0612007

Brandhuber, Spence, Zoubos, GT

0704.0245 [hep-th]

Nasti, GT

0706.0976 [hep-th]

Brandhuber, Heslop, GT

0707.1153 [hep-th]

Brandhuber, Heslop, Spence, GT

in preparation

Twistors, Strings, and Scattering Amplitudes, LMS Durham Symposium
Durham, August 2007
Outline

- **MHV diagrams** (Cachazo, Svrcek, Witten)
  - Loop MHV diagrams (Brandhuber, Spence, GT)

- **Amplitudes in pure Yang-Mills from MHV diagrams** (Brandhuber, Spence, Zoubos, GT)
  - All-plus amplitude

- **MHV amplitudes in N=4 SYM from a Wilson loop calculation at weak coupling**
  - One-loop calculation at $n$ points (Brandhuber, Heslop, GT)
  - Higher loops (Brandhuber, Heslop, Spence, GT, in preparation)
Motivations

- Unifying theme is **simplicity** of amplitudes
  - Geometry in **Twistor Space**
- unexplained by **Feynman diagrams**
  - Parke-Taylor formula for **Maximally Helicity Violating** amplitude of gluons (helicities are a permutation of $- - ++ ....+$)
- New methods account for this **simplicity**, and allow for very **efficient calculations**
Number of Feynman diagrams for scattering $gg \rightarrow ng$:

<table>
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<th>$n$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<td>220</td>
<td>2485</td>
<td>34300</td>
<td>559405</td>
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</table>

Result is: $\mathcal{A}(1^\pm, 2^+, \ldots n^+) = 0$

$\mathcal{A}_{\text{MHV}}(1^+ \ldots i^- \ldots j^- \ldots n^+) = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$

Large numbers of Feynman diagrams combine to produce unexpectedly and mysteriously simple expressions.
LHC is coming!
Amplitudes

\[ \mathcal{A} = \mathcal{A}(\{\lambda_i, \tilde{\lambda}_i, ; h_i\}) \]

- Colour-ordered partial amplitudes
  - momenta and polarisation vectors expressed in terms of spinors and helicities
  - colour indicesstripped off

- Planar theory
Simplicity of amplitudes persists at loop level:

- \( n \)-point MHV amplitude in N=4 SYM at one loop:

\[
\mathcal{A}_{\text{MHV}}^{1-\text{loop}} = \mathcal{A}_{\text{MHV}}^{\text{tree}} \sum
\]

- Sum of two-mass easy box functions, all with coefficient 1
• Computed in 1994 by Bern, Dixon, Dunbar, Kosower using unitarity

• Rederived in 2004 with loop MHV diagrams...
  (Brandhuber, Spence, GT)

• ...and, more recently, with a weakly-coupled Wilson loop calculation, with the Alday-Maldacena polygonal contour (Brandhuber, Heslop, GT)
All-loop conjecture of Bern, Dixon, Smirnov

Zvi Bern and Anastasia Volovich’s talks

- \( n \)-point MHV amplitudes in \( \text{N=4 SYM} \)

\[
\mathcal{M}_n = \exp \left[ \sum_{L=1}^{\infty} a^L \left( f^{(L)}(\varepsilon) \mathcal{M}_n^{(1)}(L\varepsilon) + C^{(L)} + E^{(L)}_{n}(\varepsilon) \right) \right]
\]

- \( \mathcal{M}_n := \mathcal{A}_{n,MHV} / \mathcal{A}_{n,MHV}^{\text{tree}} \)

- \( \mathcal{M}_n^{(1)}(\varepsilon) \) is the all-orders in \( \varepsilon \) one-loop amplitude

- \( f^{(L)}(\varepsilon) = f^{(L)}_0 + \varepsilon f^{(L)}_1 + \varepsilon^2 f^{(L)}_2 \)

  | anomalous dimension of twist-two operators at large spin |

- \( C^{(L)}, E^{(L)}_{n}(\varepsilon) \)

More on this later...
Another intriguing, simple amplitude:

- **All-plus amplitude in pure Yang-Mills, 1 loop**

\[
\mathcal{A}_{n}^{1-\text{loop}}(1^{+}, \ldots, n^{+}) = \frac{-i}{48\pi^2} \sum_{1 \leq l_1 < l_2 < l_3 < l_4 \leq n} \frac{\text{Tr}(\frac{1-\gamma^5}{2} \hat{l}_1 \hat{l}_2 \hat{l}_3 \hat{l}_4)}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}
\]

- like MHV amplitude, **no multiparticle poles**
- **all-plus equivalently computed in Self-Dual Yang-Mills**
- vanishes in **supersymmetric theories**
- **dimension shifting relations** (Bern, Dixon, Dunbar, Kosower)

- **Escapes naive application of MHV rules!**

One-loop vertex?
Amplitudes in Twistor Space
(Witten, 2003)

• Scattering amplitudes are supported on algebraic curves in Penrose’s twistor space

• $d = q - 1 + l$  
  $q = \#$ negative helicity gluons,  
  $l = \#$ loops

• $g \leq l$

  ‣ Tree MHV: $q=2, l=0 \Rightarrow d=1, g=0$ (complex line)
Amplitude  Twistor space structure  MHV diagrams

MHV

nMHV

nnMHV
Why MHV diagrams

- MHV amplitudes $\rightarrow$ complex lines in twistor space (Witten)
- Line in twistor space $\rightarrow$ point in Minkowski space (Penrose)
- MHV amplitude $\rightarrow$ local interaction in spacetime! (Cachazo, Svrcek, Witten)
  - Locality in lightcone formulation (Mansfield; Gorsky & Rosly)
MHV Rules
(Cachazo, Svrcek, Witten)

• MHV amplitude $\rightarrow$ MHV vertex

• Off-shell continuation for internal (possibly loop) momenta needed
  
  ‣ Same as in lightcone Yang-Mills

  ✷ Internal momentum is off-shell
  ✷ Need to define spinor $\lambda$ for an off-shell vector!

• Scalar propagators connect MHV vertices
• Off-shell prescription:

\[ L_{a\dot{a}} = l_{a\dot{a}} + z \eta_{a\dot{a}} \]

› \( l_{a\dot{a}} := l_{a\tilde{\dot{a}}} \) is the off-shell continuation

› \( \eta \) is a reference vector
• Draw all diagrams obtained by sewing
  \[ d = q - 1 + l \]
  MHV vertices

  \[ q = \# \text{ negative helicity gluons}, \]
  \[ l = \# \text{ loops} \]

• Examples:
  - MHV: \( q=2, \ l=1 \quad d=2 \)
  - All minus: \( q=n, \ l=1 \quad d=n \)
  - All plus: \( q=0, \ l=1 \quad d=0 \ ??? \)
One-loop MHV amplitudes in N=4

(Brandhuber, Spence, GT)

\[ \sum \int dM \]

• **Sum** over
  - all possible MHV diagrams
  - internal particle species \((g, f, s)\) and helicities

• \(dM = \text{phase space measure} \times \text{dispersive measure}\)

• **Different from unitarity-based approach** of BDDK
The integration measure

- $P_L$ is the momentum on the left

\[ d\mathcal{M} := \frac{d^4L_1}{L_1^2 + i\epsilon} \frac{d^4L_2}{L_2^2 + i\epsilon} \delta^{(4)}(L_2 - L_1 + P_L) \]

- Use $L = l + z\eta$, and $L \rightarrow (l, z)$

\[ \frac{d^4L}{L^2 + i\epsilon} = \frac{dz}{z + i\text{sgn} (l_0\eta_0)\epsilon} \frac{d^3l}{2l_0} \]

dispersive measure $\times$ phase-space measure (Nair measure)
Applications (with supersymmetry)

- One-loop MHV amplitudes in $\mathbf{N}=4$ SYM
  (Brandhuber, Spence, GT)

- One-loop MHV amplitudes in $\mathbf{N}=1,2$ SYM
  (Bedford, Brandhuber, Spence, GT; Quigley, Rozali)
  - No twistor string theory for $\mathbf{N}=1$ SYM, nevertheless MHV diagram method works
Proving MHV diagrams at one loop
Supersymmetric theories

(Brandhuber, Spence, GT)

• Covariance \((\eta\text{-independence})\)
  Feynman Tree Theorem

• Correct singularity structure
  ‣ Discontinuities across (generalised) cuts
  ‣ Soft, collinear
  ‣ Multiparticle
Use tree-level BCFW proof at one loop:

- If all singularities match, and the amplitude is covariant, then $\mathcal{A}_{\text{MHV}} - \mathcal{A}_{\text{Feynman}}$ is a polynomial in the external momenta whose dimension is $4 - \# \text{ particles}$

$$\mathcal{A}_{\text{MHV}} = \mathcal{A}_{\text{Feynman}}$$
• Proof from field redefinition on lightcone Yang-Mills action (Mansfield)

• Proof from twistor actions (Boels, Mason, Skinner)

Tim Morris and Rutger Boels talks tomorrow

• Relation with BCFW recursion relation (tree level) (Risager)
Without supersymmetry

- **Cut-constructible part of one-loop MHV amplitudes in pure Yang-Mills**
  (Bedford, Brandhuber, Spence, GT)

- **Rational terms in non-supersymmetric amplitudes missed by MHV diagrams**
  - Non-supersymmetric amplitudes are not cut-constructible in four dimensions
  - Use recursive techniques to derive rational terms
    (Bern, Dixon, Kosower; Bern, Berger, Dixon, Forde, Kosower)
The all-minus amplitude
(Brandhuber, Spence, GT)

- $n$ three-point MHV vertices (for $A(1^- \cdots n^-)$)

- Key observation: three-point MHV vertices are the same as lightcone vertices $\Rightarrow$ result is a priori correct
Explicit calculation

• Use supersymmetric decomposition:

\[ A_g = (A_g + 4A_f + 3A_s) - 4(A_f + A_s) + A_s \]

• \(N=4\) and \(N=1\) contributions vanish

• Gluon \(\rightarrow\) scalar running in the loop
  
  ▶ simpler to calculate
\[ K_4 = -\epsilon (1 - \epsilon) I_4^{D=8-2\epsilon} \xrightarrow{\epsilon \to 0} -\frac{1}{6} \]

(Originally derived by Bern & Kosower, and Bern & Morgan)
Finiteness of the all-minus amplitude

- Define $L_D = L_4 + L_{-2\varepsilon}$ with $L_D^2 = L_4^2 + L_{-2\varepsilon}^2 := L_4^2 - \mu^2$

- A finite, non-zero result arises from incomplete cancellations of propagators

\[
\frac{L_4^2}{L_D^2} = \frac{L_4^2 - \mu^2 + \mu^2}{L_D^2} = 1 + \frac{\mu^2}{L_D^2}
\]

- MHV vertices are 4-dimensional

- D-dimensional propagators
• Naive calculation directly in 4d gives zero

• Finite, non-zero result related to an anomaly?

  ▶ Finiteness arises as $\varepsilon/\varepsilon$ effect

  ▶ Anomaly in worldsheet conformal symmetry in $N=2$ open strings (Chalmers, Siegel)
• **All-minus** amplitude understood within MHV diagram method

• **All-plus** amplitude
  
  ‣ Parity conjugate of all-minus, but MHV method treats the two helicities differently

• Longstanding speculations on a one-loop all-plus vertex
  
  ‣ All-plus amplitude has **no multiparticle poles** (as MHV)
  
  ‣ Twistor space geometry seems to confirm this
Where is the all-plus amplitude?

Go back to the path integral!

- **Mansfield's procedure:** (in a nutshell)
  - Start from lightcone quantisation of YM, \( A^- = 0 \)
  - Integrate out \( A^+ \) (no derivatives wrt lightcone time \( x^- \))
  - \( A_z, A_{\bar{z}} \) correspond to physical polarisations

- **Action is** \( S = S^{--+} + S^{--+} + S^{+++} + S^{---++} \)

(Scherk, Schwarz)

anti MHV
• Change variables in path integral: \( A_z, \bar{A}_{\bar{z}} \rightarrow B_+, B_- \)

\[
(S^{-+} + S^{-++})[A_z, A_{\bar{z}}] = S^{-+}[B_+, B_-]
\]

• LHS is SDYM action
  • Bäcklund transformation

• Further require:
  
  ‣ Transformation is canonical, with \( A_{\bar{z}} = A_{\bar{z}}[B_+] \)
  
  ‣ Canonicality \( \xrightarrow{\text{}} \) Jacobian equal to \( 1 \) (classically)
  
  ‣ Subtleties related to \( \det \partial_+ \)
• Plug

\[ A_z \sim B_+ + B_+^2 + B_+^3 + \cdots \]

\[ A_{\bar{z}} \sim B_- (1 + B_+ + B_+^2 + B_+^3 + \cdots) \]

in

\[(S^{--+} + S^{--++})[A_z, A_{\bar{z}}]\]

• Result is

\[ S[B_+, B_-] = S^{++} + S^{--} + S^{--++} + S^{--+++} + \cdots \]

• Vertices have MHV helicity configuration
• **Jacobian** for $A_z, A_{\bar{z}} \rightarrow B_+, B_-$ is 1 (classically)

• **Equivalence Theorem:**
  - Green’s functions of the B fields are different from those of the A fields, however
  - S-matrix elements are the same modulo a wave-function renormalisation...
    - ...equal to 1 at one loop (Ettle & Morris)

• We can equivalently calculate amplitudes with B fields insertions
One missing thing!

- We have just mapped Self-Dual Yang-Mills to a free theory...
- ...with the consequence of eliminating the all-plus amplitude

- Potential sources of problems:
  - Jacobian
  - Equivalence Theorem
  - Regularisation
Our solution

(Brandhuber, Spence, Zoubos, GT)

• Use **Thorn worldsheets friendly regularisation**
  ‣ inherently four-dimensional

• Perform Mansfield-Bäcklund transformation on the **regularised, 4d action**
  ‣ SDYM classically integrable only in 4d

• **New one-loop effective interactions from regularisation**, plus

• **Usual MHV vertices**

```plaintext
simple...
```
- **Worldsheet friendly regulator:**
  \[ \exp(-\delta \sum_{i=1}^{n} q_i^2) \]
  \[ q^2 = 2q_z q_{\bar{z}} \]
  - \( \delta \) is sent to zero at the end of calculation

- **\( q_i \) are loop region (T-dual) momenta**

- **Regularisation generate Lorentz-violating processes**
  - cancel with appropriate ++ counterterm
  (Chakrabarti, Qiu, Thorn)
• Applying Mansfield transformation on counterterm generates all-plus amplitudes:

\[ \text{counterterm} \sim \frac{g^2 N}{12\pi^2} \left( (k_\perp^2 + (k_\perp')^2 + k_\perp k_\perp') \right) \]

Reminders: \( A = A(B) \) holomorphic

\( A \) is positive-helicity gluon

Equivalence Theorem: \( A \rightarrow B \)

Counterterm acts as a generating functional of all-plus amplitudes
• Explicit check at four points

• Soft, collinear limits
A complementary solution
(Ettle, Fu, Fudger, Mansfield, Morris)

- Use **dimensional regularisation**
  - new interactions due to the regularisation
  - vanish as $\varepsilon \to 0$
- Perform Mansfield-Bäcklund transformation on the **full D-dimensional action**
- **Violations of the equivalence theorem** produce the **missing amplitudes**

Tim Morris’s talk tomorrow
Next tasks

- Calculate more general amplitudes, including rational terms
- First example: $-++.....+$
Gravity

- Simplicity of gravity amplitudes
  - Twistor space structure (Bern, Bjerrum-Bohr, Dunbar)
  - Tree-level MHV rules from recursion relations (Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager)
  - Applications to one-loop MHV diagrams (Nasti, GT)
  - Field redefinitions on lightcone gravity action (Ananth, Theisen)
  - Recursion relations (Bedford, Brandhuber, Spence, GT; Cachazo, Svrcek; Benincasa, Boucher-Veronneau, Cachazo)

- Finiteness of N=8? (Bern, Dixon, Roiban; Bern, Carrasco, Dixon, Johansson, Kosower, Roiban; Green, Russo, Vanhove)

- Surprises even without supersymmetry! (Bern, Carrasco, Forde, Ita, Johansson)
Back to \textbf{N}=4 Super Yang-Mills
Amplitudes and Wilson Loops

(Brandhuber, Heslop, GT; Brandhuber, Heslop, Spence, GT)

- We wish to calculate $< W[C] >$ at weak coupling

$$W[C] := \text{Tr} P \exp \left[ ig \oint_C d\tau \left( A_\mu(x(\tau)) \dot{x}^\mu(\tau) + \phi_i(x(\tau)) \dot{y}^i(\tau) \right) \right]$$

- Contour $C$ as in Alday-Maldacena calculation (next slide)
- When $\dot{x}^2 = \dot{y}^2$ Wilson loop is locally supersymmetric
- Choose $\dot{x}^2 = 0$ (lightlike momenta) and $\dot{y} = 0$
- In general, supersymmetry is broken globally
• Contour $C$ in the strong-coupling calculation of A&M

- Dictated by the momenta of the scattered gluons

Contour is closed

\[ p_i = k_i - k_{i+1} \]

$k$'s are T-dual (region) momenta

\[ \sum_{i=1}^{n} p_i = 0 \]

Contour is closed
Motivation

- **Computation of amplitudes at strong coupling** (Alday and Maldacena) (Fernando Alday’s talk)
  
  - dual to that of the area of a string ending on a lightlike polygonal loop embedded in the boundary of AdS
  
  - scattering in AdS is at fixed angle, large energy → similar to Gross-Mende calculation
  
  - leads to an exponential of classical string action
  
  - calculation in the T-dual variables is equivalent to that of a lightlike Wilson loop at strong coupling (Maldacena; Rey and Yee)
• Calculate $< W[C] >$ at weak coupling for $n$ points

  ‣ One loop (two-loop calculation in preparation)
  ‣ Four-point case addressed by Drummond, Korchemsky, Sokatchev

• Result: $< W[C] >$ gives the $n$-point MHV amplitude in $\mathcal{N}=4$ SYM! (modulo tree-level prefactor)

• Conjecture that equality $< W[C] > = \mathcal{M}$ persists at higher loops
Calculation done (almost) instantly. **Two classes of diagrams:**

- Gluon stretched between two segments meeting at a cusp
  - A. Infrared divergent

- Gluon stretched between two non-adjacent segments
  - B. Infrared finite
• Clean separation between infrared-divergent and infrared-finite terms

  ▪ Important advantage, as $\varepsilon$ can be set to zero in the finite parts from the start

• From diagrams in class A:

\[
M_n^{(1)} |_{IR} = -\frac{1}{\varepsilon^2} \sum_{i=1}^{n} \left( \frac{-s_{i,i+1}}{\mu^2} \right)^{-\varepsilon}
\]

  ▪ $s_{i,i+1} = (p_i + p_{i+1})^2$ is the invariant formed with the momenta meeting at the cusp
• Diagram in class B, with gluon stretched between \( p \) and \( q \) gives a result proportional to

\[
F_{\varepsilon}(s, t, P, Q) = \int_0^1 d\tau_p d\tau_q \frac{P^2 + Q^2 - s - t}{[-(P^2 + (s - P^2)\tau_p + (t - P^2)\tau_q + (-s - t + P^2 + Q^2)\tau_p\tau_q)]^{1+\varepsilon}}
\]

• Explicit evaluation shows that this is equal to the finite part of a 2-mass easy box function:
\[
F_m(s, t, P^2, Q^2) = -c! 2^{-2}\left[ (1 - e^2)^{-2} F_1(1, -e^2, \ldots, -e^2, aP^2) - (1 - e^2)^{-2} F_1(1, -e^2, 1 - e^2, aQ^2) \right]
\]

where

\[
s := (p + P)^2
\]

\[
t := (q + P)^2
\]

In the example: \( p = p_2 \quad q = p_5 \)

\[
P = p_3 + p_4 , \; Q = p_6 + p_7 + p_1
\]

- One-to-one correspondence between Wilson loop diagrams and finite parts of 2-mass easy box functions
- Explains why each box function appears with coefficient equal to 1 in the expression of the one-loop N=4 MHV amplitude
Explicit calculation gives:

\[ F_\varepsilon = -\frac{1}{\varepsilon^2} \cdot \left[ \left( \frac{a}{1 - aP^2} \right)^\varepsilon \ _2F_1 \left( \varepsilon, \varepsilon, 1 + \varepsilon, \frac{1}{1 - aP^2} \right) + \left( \frac{a}{1 - aQ^2} \right)^\varepsilon \ _2F_1 \left( \varepsilon, \varepsilon, 1 + \varepsilon, \frac{1}{1 - aQ^2} \right) \right. \]

\[ \left. - \left( \frac{a}{1 - as} \right)^\varepsilon \ _2F_1 \left( \varepsilon, \varepsilon, 1 + \varepsilon, \frac{1}{1 - as} \right) - \left( \frac{a}{1 - at} \right)^\varepsilon \ _2F_1 \left( \varepsilon, \varepsilon, 1 + \varepsilon, \frac{1}{1 - at} \right) \right] \]

At \( \varepsilon \to 0 \):

\[ F_{\varepsilon=0} = -\text{Li}_2(1 - as) - \text{Li}_2(1 - at) + \text{Li}_2(1 - aP^2) + \text{Li}_2(1 - aQ^2) \]

- Box function in the same compact form derived from dispersion integrals using one-loop MHV diagrams
  
  (Brandhuber, Spence, GT)
At 4 points, all-orders in $\varepsilon$ result:

\[ M_4^{(1)}(\varepsilon) = -\frac{2}{\varepsilon^2} \left[ \left( \frac{-s}{\mu^2} \right)^{-\varepsilon} {}_2F_1 \left( 1, -\varepsilon, 1 - \varepsilon, 1 + \frac{s}{t} \right) + \left( \frac{-t}{\mu^2} \right)^{-\varepsilon} {}_2F_1 \left( 1, -\varepsilon, 1 - \varepsilon, 1 + \frac{t}{s} \right) \right] \]

- Agrees with result of Green, Schwarz and Brink

For $n > 4$, missing topologies (vanish as $\varepsilon \to 0$)

- E.g. $n > 5$, get only parity-even part
\[
\langle W[C] \rangle := 1 + \sum_{L=1}^{\infty} a^L W^{(L)} = \exp \sum_{L=1}^{\infty} a^L w^{(L)}
\]

- Key result: non-abelian exponentiation theorem (Gatheral; Frenkel and Taylor)

- \( w \)'s are calculated by keeping only terms containing maximal non-abelian colour factor
  
  - subset of all possible diagrams
• BDS’s Exponential Ansatz naturally emerges

\[ \mathcal{M}_n := 1 + \sum_{L=1}^{\infty} a^L \mathcal{M}^{(L)}_n = \exp \left[ \sum_{L=1}^{\infty} a^L \left( f^{(L)}(\epsilon) \mathcal{M}^{(1)}_n(L\epsilon) + C^{(L)} + E_n^{(L)}(\epsilon) \right) \right] \]

\[ \langle W_n[C] \rangle := 1 + \sum_{L=1}^{\infty} a^L W^{(L)}_n = \exp \sum_{L=1}^{\infty} a^L w^{(L)}_n \]

• If \( < W[C] > = \mathcal{M} \), then

\[ w^{(L)}_n = f^{(L)}(\epsilon) \mathcal{M}^{(1)}_n(L\epsilon) + C^{(L)} + O(\epsilon) \]
• Calculation of \( w \) at \textbf{two loops} almost completed \textit{Stay tuned!}

• Four-point MHV amplitude fixed using \textbf{dual conformal invariance} and \textbf{factorisation of infrared divergences} (Drummond, Korchemsky, Sokatchev)
  - appears to be not restrictive enough for \( n > 4 \)
  - issues with \textbf{anomalous dimension} of \textbf{twist 2} operators
Summary

• **Simplicity** of scattering amplitudes in *Twistor Space*

• New, efficient methods to derive amplitudes
  ‣ MHV diagrams
  ‣ recursion relations, generalised unitarity...
• **MHV diagrams:** provide a new diagrammatic method to calculate scattering amplitudes at tree and one-loop level in super Yang-Mills

• **Progress in non-supersymmetric Yang-Mills**
  - All-minus amplitude, all-plus amplitude
  - 4d Mansfield-Bäcklund transformation
  - worldsheet friendly regularisation
• MHV amplitude in N=4 SYM from a Wilson loop calculation at weak coupling
  ‣ One loop
  ‣ Higher loops
Some of the pressing questions...

- Rational terms in pure YM amplitudes
- Higher loops
- Relation to integrability
- Wilson loop calculations to higher loops
- What about correlators of gauge-invariant operators?

...and many more...