DURHAM UNIVERSITY
Department of Mathematical Sciences

Level 1 Mathematics modules
Course Booklet
2014 - 2015

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Web: www.durham.ac.uk/mathematical.sciences
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1 General Information

Welcome to the Department of Mathematical Sciences! 1,200 undergraduates take modules provided by the Department. This booklet provides information on first-year modules offered by the department. It also contains summary information on key policies related to assessment and academic progress.

Full details of the department’s policies and procedures are available in the departmental degree programme handbooks at http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/, which also contains on on-line version of the course descriptions contained in this booklet.

Information concerning general University regulations, examination procedures etc., are contained in the Faculty Handbooks (www.dur.ac.uk/faculty.handbook) and the University Calendar, which provide the definitive versions of University policy. The Teaching and Learning Handbook (www.dur.ac.uk/teachingandlearning.handbook) contains information about assessment procedures, amongst other things.

You should keep this booklet for future reference. For instance, prospective employers might find it of interest. You can look forward to an enjoyable year.

1.1 Useful Contacts

The first point of contact for issues referring to a particular course or module should be the relevant lecturer. For more general questions or difficulties you are welcome to consult the Course Director or your Adviser. For queries relating to teaching issues, for example registration, timetable clashes, support for disabilities or illness, you should visit the department to speak to someone in the main Maths Office (CM201), or send an email to maths.teaching@durham.ac.uk.

Head of Department:
maths.head@durham.ac.uk

Director of Undergraduate Studies:
Dr Peter Bowcock (CM307, peter.bowcock@durham.ac.uk)

The Course Directors for students are determined by their programme and level of study as follows:

Students on Mathematics programmes at level one:
maths.1hcoursedirector@durham.ac.uk

Students on Mathematics programmes at level two:
maths.2hcoursedirector@durham.ac.uk

Students on Mathematics programmes at levels three and four:
maths.34hcoursedirector@durham.ac.uk

Students on Natural Sciences and Combined Honours programmes at all levels:
maths.natscidirector@durham.ac.uk

Students on programmes other than Mathematics and Natural Sciences and Combined Honours at all levels:
maths.otherprogdirector@durham.ac.uk

We may also wish to contact you! Please keep the Mathematics Office informed of your current term-time residential address and e-mail address.
1.2 Course Information

Term time in Durham is Michaelmas (10 weeks), Epiphany (9 weeks) and Easter (9 weeks). There are 22 teaching weeks, and the last seven weeks are dedicated to private revision, examinations and registration for the subsequent academic year.

Timetables giving details of places and times of your commitments are available on Departmental web pages and noticeboards in the first floor corridor of the Department. It is assumed that you read them!

You can access your own Maths timetable at www.maths.dur.ac.uk/teaching/ and then clicking on the ‘My Maths timetable’ link.

Also, teaching staff often send you important information by e-mail to your local '@durham.ac.uk' address, and so you should scan your mailbox regularly.

Note that in October it takes time to sort out groups for tutorials and practicals, and so these classes start in week 2.

1.3 Assessment

Full details of the University procedures for Examinations and Assessment may be found in Section 6 of the Learning and Teaching Handbook, http://www.dur.ac.uk/learningandteaching.handbook/. The Department’s policies and procedures are described in the departmental degree programme handbook, http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/. The Department follows the marking guidelines set out by the University Senate:

<table>
<thead>
<tr>
<th>Degree Class</th>
<th>Marking Range(%)</th>
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</thead>
<tbody>
<tr>
<td>I</td>
<td>70 – 100</td>
</tr>
<tr>
<td>II(i)</td>
<td>60 – 69</td>
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<tr>
<td>II(ii)</td>
<td>50 - 59</td>
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<tr>
<td>III</td>
<td>40 – 49</td>
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<tr>
<td>Fail</td>
<td>0 – 39</td>
</tr>
</tbody>
</table>

Linear Algebra I (MATH1071), Calculus & Probability I (MATH1061) and Analysis I (MATH1051) are assessed by written examination. For Programming & Dynamics I (MATH1041), 40% of the assessment is based on summative coursework submitted in the programming part of the module and 60% is based on a written examination on the Dynamics part of the module. For Discrete Maths(MATH1031), 40% is based on summative work submitted during the Epiphany term (a presentation and a written report) and 60% is based on a written examination. For all other first year modules offered by the department, 10% of the assessment is based on summative coursework and 90% is on a written examination.

All courses include either summative or formative assessed work, with assignments being set on a regular basis in lecture-based courses. The purpose of formative and summative assessment of coursework is to provide feedback to you on your progress and to encourage effort all year long.
Regular assignments are marked A-E to the following conventions:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Equivalent Mark</th>
<th>Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>≥ 80%</td>
<td>Essentially complete and correct work</td>
</tr>
<tr>
<td>B</td>
<td>60%—79%</td>
<td>Shows understanding, but contains a small number of errors or gaps</td>
</tr>
<tr>
<td>C</td>
<td>40%—59%</td>
<td>Clear evidence of a serious attempt at the work, showing some understanding, but with important gaps</td>
</tr>
<tr>
<td>D</td>
<td>20%—39%</td>
<td>Scrappy work, bare evidence of understanding or significant work omitted</td>
</tr>
<tr>
<td>E</td>
<td>&lt;20%</td>
<td>No understanding or little real attempt made</td>
</tr>
</tbody>
</table>

**Use of Calculators in Exams** The use of electronic calculators is allowed in some module examinations and other module assessments. Each student taking modules offered by departments or schools within the science faculty, which specify that calculators be allowed in assessments, will be offered a calculator, free of charge, at the beginning of their course. The model will be a Casio fx-83 GTPLUS or a Casio fx-85 GTPLUS.

Calculators will become the property of students who will be responsible for their upkeep. No replacement calculators will be provided free of charge, but may be available to purchase from departments/schools, depending on availability. The specified calculator will also be generally available, in shops and online, should a replacement purchase be necessary.

Where the use of calculators is allowed in assessments, including examinations, the only models that will be allowed are either a Casio fx-83 GTPLUS or a Casio fx-85 GTPLUS. In particular, examination invigilators will refuse to allow a candidate to use any calculator other than the model(s) specified, which will be explicitly stated on the front of the examination paper. During examinations no sharing of calculators between candidates will be permitted, nor will calculators or replacement batteries be supplied by the Department or the Student Registry Office.

**1.4 Academic progress**

The Department is responsible for ensuring that students are coping with the courses and meeting their academic commitments.

For 1st year modules you are required:
- to attend tutorials/ problems classes/ computer practical classes
- to sit collections exams
- to submit summative or formative assessed work on time to a satisfactory standard.

Assessed work which is graded D or E is counted as being of an unsatisfactory standard.

Attendance and submission of work is monitored through a database. It is your responsibility to ensure that your attendance is recorded by signing the relevant attendance sheets.

Students who are not keeping up with their commitments will be contacted by course directors to help get them back on track.

Persistent default will result in a formal written warning, which may be followed by the initiation of Faculty procedures.

Full details of academic progress requirements for the department are available in the departmental degree programme handbook, [http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/](http://www.dur.ac.uk/mathematical.sciences/teaching/handbook/).
MathSoc: Necessary and Sufficient

Durham University Mathematical Society, affectionately known as MathSoc, provides an opportunity for maths students (or anyone with an interest in maths) to meet away from lectures.

We arrange a variety of events throughout the year, including bar crawls, talks by guest speakers, a Christmas meal, and the highlight of the year – a trip to see Countdown being filmed! So there’s something for everyone. We are currently sponsored by EY so we are able to offer many exclusive networking opportunities and careers events!

MathSoc works with the Maths Department to arrange Undergraduate Colloquia, where departmental and external lecturers give relaxed, inspiring talks on their current research. These cover a wide range of mathematical topics with previous titles including 'Dot-dots, zig-zags and plank-planks' and 'Defects of integrable field theory'. These are at a level such that anyone with an interest in maths can enjoy them and they aim to motivate an interest in an area of maths you may not have seen before.

We have our own website (durhammathsoc.wix.com/mathematicalsociety), where you will find all the most up-to-date information about the society. Here you will also find our second-hand book list, which has many of the books needed for courses for much cheaper than you will find them in the shops. Last year people saved up to £50 by using this service!

If you would like any more information about either the society itself, or advice on any other aspect of the maths course, please do not hesitate to get in touch with any of our friendly exec members listed below or via the society email address (mathematical.society@durham.ac.uk). You can also find us on Facebook by searching for ”The Official Durham MathSoc Group”.

To join:

Come and see our stand at the freshers’ fair, or email us at any time: it costs only £7 for life membership, or £4 for a year. You can sign up on the Durham Students Union Website (http://www.durhamsu.com/groups/mathematical).

This year’s Exec is:

<table>
<thead>
<tr>
<th>Role</th>
<th>Name</th>
<th>Email</th>
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<tbody>
<tr>
<td>President</td>
<td>Jane Robinson</td>
<td><a href="mailto:jane.robinson@durham.ac.uk">jane.robinson@durham.ac.uk</a></td>
</tr>
<tr>
<td>Secretary</td>
<td>Jack Tellyn</td>
<td><a href="mailto:j.o.tellyn@durham.ac.uk">j.o.tellyn@durham.ac.uk</a></td>
</tr>
<tr>
<td>Treasurer</td>
<td>Daniel McKenna</td>
<td><a href="mailto:d.o.mckenna@durham.ac.uk">d.o.mckenna@durham.ac.uk</a></td>
</tr>
<tr>
<td>Social Secretary</td>
<td>Gina Cuomo</td>
<td><a href="mailto:g.m.cuomo@durham.ac.uk">g.m.cuomo@durham.ac.uk</a></td>
</tr>
<tr>
<td>Publicity Officer</td>
<td>Eleanor Kershaw-Green</td>
<td><a href="mailto:e.l.kershaw-green@durham.ac.uk">e.l.kershaw-green@durham.ac.uk</a></td>
</tr>
</tbody>
</table>
1.6 Disclaimer

The information in this booklet is correct at the time of going to press in May 2014. The University, however, reserves the right to make changes without notice to regulations, programmes and syllabuses. The most up-to-date details of all undergraduate modules can be found in the Faculty Handbook on-line at www.dur.ac.uk/faculty.handbook/.

1.7 Booklists and Descriptions of Courses

The following pages contain brief descriptions of the Level 1 courses in Mathematics. The core modules Linear Algebra I, Calculus & Probability I, Analysis I and Problem Solving & Dynamics I are compulsory for Mathematics students, and you may also choose to take one or two of the optional modules Discrete Mathematics and Statistics. Supporting the core modules there is the optional “Brush Up Your Skills” weekly course.

The other three modules offered - Single Mathematics A, Single Mathematics B and Mathematics for Engineers and Scientists - are not open to students on Mathematics degrees, but will be of interest to Natural Science students or students in other departments who want to take a Level 1 Mathematics module. Note that these modules will not allow you to progress to any Level 2 or higher Mathematics modules.

These descriptions supplement the official descriptions in the module outlines in the faculty handbook which can be found at http://www.dur.ac.uk/faculty.handbook/module_search/?search_dept=MATH&search_level=1. Note that the official module outlines contain information on module pre- and co-requisites, excluded combinations, assessment methods and learning outcomes. The descriptions which follow supplement this by providing a list of recommended books and a brief syllabus for each module.

For some modules you are advised to buy a particular book, indicated by an asterisk; for others a choice of titles is offered or no specific recommendation is given. There are also suggestions for preliminary reading and some time spent on this during the summer vacation may well pay dividends in the following years.

Syllabuses, timetables, handbooks, exam information, and much more may be found at www.maths.dur.ac.uk/teaching/ or by following the link ‘teaching’ from the Department’s home page (www.maths.dur.ac.uk). These syllabuses are intended as guides to the modules. The definitive information on course content and expected learning outcomes is in the official module outlines.
Calculus is a fundamental part of mathematics and provides a foundation for all your future mathematical studies. This course will seek to consolidate and expand your knowledge of this topic and is designed to be completely accessible to the beginning calculus student. The three basic concepts of calculus will be covered, namely, limits, differentiation and integration. The emphasis of this module is on concrete methods for calculation, while the Analysis I module will revisit the above concepts and provide a deeper knowledge with a more formal approach.

First and second order ordinary differential equations are studied together with solution methods that are naturally associated with the techniques of integration. Taylor and Fourier series are also covered, in preparation for their application in later modules. Numerous exercises are provided to reinforce the material.

**Recommended Books**


Both these books are useful in several modules at level 1 and 2 (Analysis I, Problem Solving and Dynamics I, Mathematical Physics II, Analysis in Many Variables II). All mathematicians have to understand calculus, so there are many books aimed at this vast market and a wide selection can be found in the University library. A particularly concise book that might appeal to some students is


**Preliminary Reading**: Revise A-level Core Mathematics material in your favourite books.

**Calculators** Electronic calculators are not permitted in this examination.
Outline of course

**Aim**: To master a variety of methods for solving problems and acquire some skill in writing and explaining mathematical arguments. To develop probabilistic insight and computational skills.

**Term 1** (30 lectures)

**Elementary Functions of a Real Variable**: Domain and range. Graphs of elementary functions. Even and odd functions. Exponential, trigonometric and hyperbolic functions. Algebraic combinations and composition. Injective, surjective and bijective functions. Theorem of inverse functions. Logarithm function as inverse of exponential function; inverse trigonometric functions.


**Integration**: Antiderivatives. Fundamental theorem of calculus. Integration by parts and use of partial fractions to integrate rational functions. Integration of even/odd functions. Gaussian integration.

**Ordinary Differential Equations**: First order: separable, exact, homogeneous, linear. Second and higher order: linear with constant coefficients, importance of boundary conditions, reduction to a set of first order equations, treatment of homogeneous and inhomogeneous equations, particular integral and complementary function.

**Taylor’s Theorem**: Taylor polynomials. Statement of Taylor’s theorem with Lagrange remainder. Taylor series expansions of \( e^x, \sin x, \sinh x, \log(1 + x) \).

**Functions of several variables**: Continuity. Partial differentiation. Chain rule. Taylor polynomial in two variables.

**Fourier Series**: Orthogonal functions and Fourier series. Convergence, periodic extension, sine and cosine series, half-range expansion. Parseval’s theorem.

**Multiple Integration**: iterated sums, double and triple integrals by repeated integration, volume enclosed by surface, Jacobians and change of variables.
1.7.2 CALCULUS AND PROBABILITY I – MATH1061
TERM 2: PROBABILITY (22 lectures)

Dr M. C. M. Troffaes

Probability is a concept with applications in all numerate disciplines e.g. in mathematics, science and technology, medicine, engineering, agriculture, economics and many other fields. In this course, the theory of probability is developed with the calculus and analysis available and with applications in mind. Among the topics covered are: probability axioms, conditional probability, special distributions, random variables, expectations, generating functions, applications of probability, laws of large numbers, central limit theorems.

Recommended Books

The following book is very good:


The DUO site will provide information about some other textbooks.

A lot of information is available from the website en.wikipedia.org/wiki/Probability

Calculators Electronic calculators are not permitted in this examination.
Outline of course

**Aim**: to develop probabilistic insight and computational skills.

**Term 2** (22 lectures)

**Introduction to probability**: chance experiments, sample spaces, events, assigning probabilities. Probability axioms and interpretations.

**Conditional probability**: theorem of total probability, Bayes theorem, independent events. Applications of probability.

**Random variables**: discrete probability distributions and distribution functions, binomial, Poisson, Poisson approximation to binomial, transformations of random variables. Continuous random variables: probability density functions, normal distribution, normal approximation to binomial.

**Joint, marginal and conditional distributions**.

**Expectations**: expectation of transformations, variance, covariance, expectations of expectations, Chebyshev’s inequality, weak law of large numbers. Moment-generating functions.

**Central-limit theorems**.
Techniques from linear algebra are used in all of mathematics. This course gives an introduction to all the major ideas in the topic. The things you learn in this course will be very useful for most modules you take later on.

The first term is concerned with the solution of linear equations and the various ways in which the ideas involved can be interpreted including those given by matrix algebra, vector algebra and geometry. This enables us to determine when a system of equations has a unique solution and gives us a systematic way of finding it. These ideas are then developed further in terms of the theory of vector spaces and linear transformations. We will discuss examples of linear transformations that are familiar from geometry and calculus.

Any linear map can be put into a particularly easy form by changing the basis of the space on which it acts. The second term begins with the solution of the eigenvalue problem which tells you how to find this basis. We then go on to generalise the notions of length, distance and angle to any vector space. These ideas may be used in a surprisingly large range of contexts. We show how all these ideas come together in the applications to geometry and calculus introduced in the first term.

Throughout the course we will also discuss examples for the notion of a group, which is one of the fundamental organizing objects in mathematics.

**Recommended Books**

- D.C. Lay, *Linear Algebra and its Applications*.
- H. Anton and R.C. Busby, *Contemporary Linear Algebra*.
- G. Strang, *Introduction to Linear Algebra*.
- T.S. Blyth, E.F. Robertson, *Basic Linear Algebra*.
- T.S. Blyth, E.F. Robertson, *Further Linear Algebra*.

**Calculators**

Electronic calculators are not permitted in this examination.
Outline of course

**Linear Algebra 1**

**Term 1** (30 lectures)

**Vectors in \( \mathbb{R}^n \)** (6 lectures) Vectors, addition and scalar multiplication in \( \mathbb{R}^n \) with concrete examples in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \). Scalar product, vector product, triple product. Equations of lines and planes, linear systems of equations in 3 variables. Examples: scalar and vector equations of lines and planes in \( \mathbb{R}^3 \)


**Determinants and Groups I** (6 lectures) Determinants and explicit methods for their calculation (row and column expansion). Properties of determinants. Axioms of groups. Examples: symmetric groups, \( \text{GL}(n) \), \( \text{SL}(n) \). The determinant in terms of permutations. Examples: areas of parallelograms, volumes of parallelepipeds.

**Vector spaces** (7 lectures) Vector spaces and subspaces over \( \mathbb{R} \). Examples: lines and planes in \( \mathbb{R}^3 \). Linear independence, spanning sets, bases and coordinates, dimension. Vector spaces of polynomials. Affine subspaces. \( \mathbb{C}^n \) as a vector space.

**Linear mappings** (6 lectures) Definition of linear mapping, matrices as linear mappings in \( \mathbb{R}^n \)(examples: dilations, projections, reflections, rotations in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \)). Differentiation and integration as a linear mapping (example: polynomials). Representation of linear mappings by matrices. Composition of linear mappings and matrix multiplication. Kernel (row and column), rank and image of a linear mapping.

**Term 2** (22 lectures & 4 problem classes plus collection)

**Change of basis and diagonalisation** (7 lectures) Change of basis and of coordinates for linear maps. Eigenvalues and eigenvectors. Explicit calculation with characteristic polynomial. Diagonalisation by change of basis.

**Inner product spaces** (8 lectures) Definition and examples: \( \mathbb{R}^n \), \( \mathbb{C}^n \), polynomials. Cauchy–Schwarz inequality. Orthonormal bases and Gram–Schmidt procedure. Orthogonal and unitary matrices. Examples: projection, reflections and distances in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \). Orthogonal complement of a subspace. Diagonalisation of symmetric matrices by orthogonal matrices.

**Linear differential operators** (3 lectures) 2nd order linear differential operators. Special polynomials as eigenfunctions.

**Groups II** (4 lectures) More examples of linear groups: \( \text{O}(n), \text{U}(n) \). Modular arithmetic: \( \mathbb{Z}_n; \mathbb{Z}_n^\times \). Matrix realisation of symmetry groups of polygons (\( \mathbb{Z}_n \), dihedral groups).
1.7.4 ANALYSIS I – MATH1051 (37 lectures)

Prof N. Peyerimhoff

This course deals mainly with ‘limits of infinite processes’. It provides a firm foundation for the operations of differentiation and integration that you already know something about. In addition, you will learn how to answer questions such as the following:

(a) What is the limit of the sequence \((2/1)^1, (3/2)^2, (4/3)^3, (5/4)^4, \ldots\) of rational numbers? [Answer: the transcendental number \(e\).]

(b) It is not hard to believe that the geometric series \(1 + 1/2 + 1/4 + 1/8 + \ldots\) converges to the value 2, but what does the series \(1 + 1/2 + 1/3 + 1/4 + \ldots\) converge to? [Answer: it does not converge.]

(c) What is the value of the integral \(\int_{0}^{\infty} \frac{x^{5/2}}{1+x^2} \, dx\)? [Answer: it does not exist.]

We shall discuss techniques for answering questions of this sort. But analysis consists of more than simply problem-solving. Ultimately, it is about constructing logical arguments (proofs), using the correct language and style, and what mathematicians call rigour. Acquiring this skill is more important than learning problem-solving tricks, but also more difficult, especially at first. We hope that by the end of the year, you will be able to invent and write out simple proofs.

Recommended Books

The course material is covered in many books on calculus or analysis that you will find in the various libraries. The book by Salas et al, recommended for several other modules, also covers most of the material in this course.

The following are standard American blockbusters, which also cover material in several other first-year courses:

The following are smaller and more specialised English-style books:
R. Maude, Mathematical Analysis, Edward Arnold, 1986
C. Clark, Elementary Mathematical Analysis, Wadsworth, 1982

Calculators

Electronic calculators are not permitted in this examination.
Outline of course

Aim: An understanding of the real and complex number systems, an introduction to series and limits and becoming familiar with the concept of continuity. To become familiar with sequences of functions, the concepts of differentiability, series and power series.

Term 1 (20 lectures)

Introduction

Basic logic and sets: Mathematical statements and connectives (AND, OR, NOT, ...), notation and basic concepts for sets, basic operations on sets and De Morgan’s rule.

Numbers: The number systems \((\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R})\) and the complex numbers \(\mathbb{C}\). \(|x| < c \iff -c < x < c\), \(|a| + |b| \geq |a+b| \geq ||a| - |b||\) for real (and complex) numbers.

More logic: Quantifiers, negation of statements.

Basics about sequences and limits: Notion of sequence, definition of \(e\), definition of limit and basic theorems (uniqueness of limits, COLT, pinching theorem).

Sup and inf: \(\mathbb{Q}, \mathbb{R}\) and the completeness axiom. Sup and inf of subsets of \(\mathbb{R}\) and of real valued functions. Relation to maxima/minima. \(\sup f + \sup g \geq \sup(f+g) \geq \sup f + \inf g\).

Proof techniques: Principle of indirect proof and contrapositive proof technique.


Functions, limits and continuity: Functions as maps between sets. Preimage of a set under a function. Limit of a function as \(x\) tends to infinity, \(\lim_{x\to\infty} x^a/e^x\), \(\lim_{x\to\infty} (\log x)/x^a\). Definition of continuity. Sum, composite of continuous functions is continuous. Intermediate Value theorem and applications. Bisection proof of max-min theorem.
Term 2 (21 lectures)

Aim: Having learnt sequences and basic properties of functions in the first term, to become familiar with sequences of functions, the concepts of differentiability, series and power series.

Sequences of functions: Pointwise and uniform convergence, continuity of limits of uniformly convergent continuous functions.


Convergence of Series: Infinite series and series as sequences; convergence, examples including $\sum n^{-a}$. Comparison test, absolute convergence theorem (absolutely convergent series are convergent), conditional convergence, rearrangements of series (Riemann series theorem), Cauchy product, ratio test, alternating sign test.

Integration: Brief discussion of Riemann sums. Fundamental theorem of calculus. $|\int f| \leq \int |f|$. Convergence of $\int_0^\infty f(x)dx$, comparison test, absolute convergence theorem, examples. Convergence of integrals with bounded range but unbounded integrand, comparison test, absolute convergence, examples. Formula for differentiation under the integral sign. (Integral test for convergence of series).

Power Series: Radius of convergence. Weierstrass test of uniform convergence, uniform convergence of power series. Term-by-term differentiation and integration with examples to show these results are not necessarily true for general (pointwise convergent) series of functions. Taylor series.
Some benefits of having programming skills are obvious: it is useful in mathematical/scientific computing, and it may boost your employment chances after graduation. However, the process of writing a computer program also develops general mathematical skills: logical reasoning, attention to details and the habit of checking one’s work. The computer is an ideal tool for this, as it rejects nonsensical constructs and syntax errors, and it provides instantaneous feedback on your work.

In this half-module, you will learn the fundamentals of programming using python (version 2.7), which is a general-purpose, high-level language that is widely used in real applications. Once mastered, however, it is straightforward to use the basic concepts in other programming languages.

**Recommended Books/Resources**

Perhaps more than other mathematics modules, programming is learnt mostly by doing (trying things out on a computer and learning from your mistakes) and very little by reading books.

Much of the what is covered in the lectures is also covered in this book, albeit at a higher level:


The authoritative manual of the python language is freely available online, and (unlike many online resources) it contains a readable tutorial section:

- Python documentation, [http://docs.python.org/2/index.html](http://docs.python.org/2/index.html)
Outline of course

Aim: basic principles and basic competence in computer programming

Term 1 (10 lectures)

Basic types (numerics), operators, variables and assignment

Control structures: conditionals, loops and functions

Floating-point arithmetic

Lists, strings and introduction to objects
Dynamics concerns evolution with time. In this course we study a model of time-development called ‘classical mechanics’. This applies to the world around us and describes the motion of everyday objects via ‘forces’. It was invented by Isaac Newton in the 17th century, when it stimulated revolutions in astronomy, physics and mathematics. Today it is a cornerstone of applied science.

This introductory course treats firstly the motion of point particles, and then the motion of a certain extended body - a flexible stretched string. Highlights include conservation laws and use of Fourier series.

We use what you have covered in Calculus I (ordinary differential equations, partial differentiation, Fourier series) and Linear Algebra I (vectors). It is vital to be familiar with this material!

The Dynamics course leads on naturally to the second-year courses ‘Mathematical Physics II’ and ‘Analysis in Many Variables II’.

**Recommended Books**


There are many other textbooks on Mechanics in the Library at shelfmarks 531, 531.1, 531.2, 531.3. eg. French & Ebison, *Introduction to Classical Mechanics*.

For vibrating strings and Fourier series, consult the relevant chapter in almost any book on ‘Mathematics for Physical Scientists’ (or Engineers). These are at Library shelfmarks 51:53, 51:54, 51:62.

**Calculators** Electronic calculators are not permitted in this examination.
Outline of course

**Aim**: to provide an introduction to classical mechanics applied to simple physical systems.

Term 2 (22 lectures)

Frames of reference, Newton’s laws in vector form, forces, mass, momentum, gravitational force, projectiles, Lorentz force and charged particles in constant electromagnetic fields.

Concepts of energy and angular momentum.

Simple harmonic motion and oscillations about a stable equilibrium. Damped oscillations and resonance.

Central forces and the use of energy and angular momentum to study planetary motion.

Waves and strings, including the derivation of the wave equation for small amplitude vibrations and its solution by separation of variables.
This module introduces a wide variety of topics about objects and structures that are discrete (like the integers) rather than continuous (like the real numbers). We will often ask ‘how many?’; these counting problems can be simple to state, using ordinary language, but surprisingly difficult to solve, needing both careful common sense and some specific techniques. We also study graphs (not the familiar graphs of functions, but networks) and their many applications.

While we will present some general mathematical techniques, many of the problems that you will encounter encourage some creative thinking in their solution; you must learn to explain your logic clearly, in some suitable combination of words, symbols and diagrams.

The main emphasis of the work of second term will be the development, via topics drawn from Graph Theory and Combinatorics, of important abilities such as problem-solving, self-study, written and oral presentation skills. This will be done through guided workshops and a presentation to other students, and will prepare you for project work in your final year. Of course these skills will be very useful for other modules, and the rest of your life.

For the first 13 weeks there are two lectures and one problems class per week. Problems are set weekly to be handed in and there is a compulsory examination (Collections) in January to see how you are going on. In May/June there is a 2-hour written examination worth 60% of the module mark. In weeks 14 to 17 and 19 there will be 2-hour workshop sessions and you will give a presentation in week 18. This presentation, and a final written report handed in at the end of week 19, are each worth 20% of the module mark.

Recommended Books

There is no required text, but any of these might be helpful or interesting:


The books by Biggs, Grimaldi and Tucker all cover most of the material. Graham, Knuth and Patashnik is a mine of interesting information and examples, written in a very chatty style. Wilson’s book is excellent for the graph theory part of the course and goes well beyond. Marcus is very good on the non-graph theory parts of the course.

Calculators

Approved electronic calculators are allowed in the examinations.
Outline of course

**Aim**: To provide students with a range of tools for counting discrete mathematical objects. To provide experience in problem-solving, presentation, mathematical writing and group working skills through guided self-study and seminars in topics in Combinatorics and Graph Theory.

**Term 1** (20 lectures)

**Principles of Counting**: Arrangements and permutations, selections and combinations, mathematical induction, combinatorial vs. computational proof. Pigeon-hole principle, inclusion-exclusion formula.

**Recurrence Relations and Generating Functions**: Recurrence relations, generating functions, partitions.

**Terms 2 & 3** (5 lectures, 5 two-hour workshops and a presentation, 2 revision lectures)

**Graphs**: Basic concepts (paths, circuits, connectedness, trees, etc.).

**Project & Presentation Topics**: Students will work through guided self-study on one of a set of Graph Theoretic or Combinatorial topics such as Planar graphs, Derangements, Graph Colouring and Optimisation problems.
1.7.8 MATHEMATICS FOR ENGINEERS AND SCIENTISTS – MATH1551 (61 lectures)

Dr S. A. Abel / Dr C. Kearton

Note: This module is not available to Mathematics students.

This module is intended to supply the basic mathematical needs for students in Engineering and other sciences.

There are revision classes during the first two weeks of term where you can practise problems and ask questions. They are based on a wide range of A-level mathematics material. The purpose is to help you brush-up on any material you have forgotten or did not cover in great detail at A-level (as not everyone has the same mathematical background.) It does not count in any way towards your final mark for this module.

There are 3 lectures each week and fortnightly tutorials. The tutorials start in Week 3. Problems will be set to be handed in each week and there is a Collection examination in December to test your understanding of the first term material. All these form an integral part of the module, and the homework is summative, constituting 10% of the final module mark. Further support is provided by way of weekly Optional Seminars. These are discussions led by a member of staff based on the material of the course that allow you to consolidate your knowledge.

Recommended Books

Students should consider buying either the two books by Stroud or the book by Stephenson.


If you are not too confident about the mathematics module then the books by Stroud will provide you with much support throughout the module. Students have found these books very helpful in previous years. You will probably already know some of the material in the first book. Stephenson is a more concise text but should also prove useful for parts of the second year mathematics module for Engineering students.

All the contents of the course are covered in e-book for engineers by Pearson, which you will be able to purchase on arrival. You may also like to refer to: (all paperbacks)

M.R.Spiegel, Vector Analysis, Schaum.

Calculators

Electronic calculators are not permitted in this examination.
Outline of course

Mathematics for Engineers and Scientists

Term 1 (28 lectures)

**Elementary Functions** (Practical): Their graphs, trigonometric identities and 2D Cartesian geometry: To include polynomials, trigonometric functions, inverse trigonometric functions, $e^x$; $\ln x$; $x^a$; $\sin(x+y)$, sine and cosine formulae. Line, circle, ellipse, parabola, hyperbola.

**Differentiation** (Practical): Definition of the derivative of a function as slope of tangent line to graph. Local maxima, minima and stationary points. Differentiation of elementary functions. Rules for differentiation of sums, products, quotients and function of a function.

**Integration** (Practical): Definition of integration as reverse of differentiation and as area under a graph. Integration by partial fractions, substitution and parts. Reduction formula, e.g. for $\int \sin^n x \, dx$.

**Complex Numbers**: Addition, subtraction, multiplication, division, complex conjugate. Argand diagram, modulus, argument. Complex exponential, trigonometric and hyperbolic functions. Polar coordinates. de Moivre’s theorem. Positive integer powers of $\sin u$; $\cos u$ in terms of multiple angles.

**Differentiation**: Limits, continuity and differentiability. L’Hopital’s rule. Leibniz rule. Newton-Raphson method for roots of $f(x) = 0$. Power series, Taylor’s and MacLaurin’s theorem, and applications.

**Vectors**: Addition, subtraction and multiplication by a scalar. Applications in mechanics. Lines and planes. Distance apart of skew lines. Scalar and vector products. Triple scalar product, determinant notation. Moments about point and line. Differentiation with respect to a scalar. Velocity and acceleration.

Terms 2 & 3 (33 lectures)


**Ordinary Differential Equations**: First order differential equations: separable, homogeneous, exact, linear. Second order linear equations: superposition principle, complementary function and particular integral for equations with constant coefficients, fitting initial conditions, application to circuit theory and mechanical vibrations.
1.7.9 SINGLE MATHEMATICS A – MATH1561 (62 lectures)

Prof W. J. Zakrzewski / Dr W. Klingenberg

Note: This module is not available to Mathematics students.

This module follows on from A-level mathematics, although many topics will be covered afresh. There are three lectures and one tutorial per week. Problems are set to be handed in each week and there is a compulsory examination (Collections) in January. These are all integral parts of the module.

It is important to do the written work conscientiously throughout the year both to prepare yourself for the examination and because there is continuous assessment for written work.

The material consists of important basic ideas and techniques in calculus and linear algebra which have applications in a huge variety of areas of science and mathematics.

Recommended Books

We will follow the content of the following book (RHB) fairly closely; see the syllabus for chapter references. K.F. Riley, M.P. Hobson and S.J. Bence, Mathematical Methods for Physics and Engineering, CUP, 3rd ed. 2006 (ISBN 9780521679718).


Calculators

Electronic calculators are not permitted in this examination.
Outline of course

Term 1 (30 lectures)

Diagnostic Test (1)


Integration [Riley Ch. 2] (10): Fundamental theorem of calculus. Natural logarithm; hyperbolic functions. Basic methods of integration including substitution, integration by parts, partial fractions, reduction formulae. Applications of integration.

Complex Numbers [Riley Ch. 3] (7): Addition, subtraction, multiplication, division, complex conjugate, modulus, argument, polar form. Argand diagram, de Moivre’s theorem, $e^{i\theta}$. Trigonometric and hyperbolic functions. Roots of unity, solutions of simple equations in terms of complex numbers, the fundamental theorem of algebra.


Terms 2 & 3 (33 lectures)

Collections exam (1)


1.7.10 SINGLE MATHEMATICS B – MATH1571 (62 lectures)

Prof R. Gregory / Dr S. A. Abel / Dr P. Heslop / Dr J. Cumming

Note: This module is not available to Mathematics students. This module follows on from A-level mathematics, although many topics will be covered afresh. There are three lectures and one tutorial per week. Problems are set to be handed in each week and there is a compulsory examination (Collections) in January. These are all integral parts of the module.

In the first term we will discuss vector algebra and some applications to mechanics and geometry, ordinary differential equations – their classification and solutions, and Fourier analysis – the representation of functions as linear superpositions of sines and cosines.

In the second and third terms we cover functions of several variables, partial differential equations, and probability. The ideas of differentiation and integration extended to functions of two or more variables give rise to partial derivatives and multiple integrals. A partial differential equation expresses a relationship involving a function of two or more variables and some of its partial derivatives. Wave motion is one of the many phenomena described by partial differential equations; an example is vibration of a stretched string, such as a guitar string. The final part of the module provides an introduction to probability.

Recommended Books


In addition:

First Term: Ordinary differential equations, vector methods and Fourier analysis can be found in most books on mathematical methods, for example:


The chapter on probability in Riley et al. covers this section of the course. The Schaum outline book S. Lipschutz, Probability provides lots of examples on fundamental concepts. An alternative, wonderful, but deeper book, which progresses to a significantly higher level, is W. Feller, Introduction to Probability Theory and its Applications, Vol. I, Wiley.

Calculators

Electronic calculators are not permitted in this examination.
Outline of course

**Term 1** (30 lectures)

**Diagnostic Test** (1)

Vectors [RHB chapter 7] (9):

Ordinary Differential Equations [RHB chapter 14] (12):

Fourier Analysis [RHB chapter 12] (8):

Terms 2 & 3 (32 lectures)

Partial differentiation [RHB chapter 5] (9):

Multiple integration [RHB chapter 6] (9):

Vector Calculus [RHB chapter 10] (8):

Probability [RHB chapter 30] (6):
Sample space, probability axioms, conditional probability, random variables, independence, probability distributions (binomial and normal distributions), expectation and variance.
Statistics attempts to make evaluations concerned with uncertainty and numerical conjectures about perplexing questions. Much of the application of mathematical modelling to the real world requires the use of statistics, and it is hence becoming ever more useful. The focus of the course is upon the understanding of real-life statistical problems. The first term’s lectures develop the basic concepts surrounding uncertainty, with an emphasis on using computer packages for exploratory data analysis. In term 2 we address mainly inferential techniques, to facilitate learning in uncertain situations.

No prior statistical knowledge is assumed. Students are required to have an A-level (with grade ’C’ at least) in a mathematics subject which may or may not be statistics, or an equivalent qualification.

There are two lectures per week and three other hours (a mixture of tutorials and computer practicals) per fortnight. Problems are set weekly to be handed in for assessment. There will be a Collection examination in January.

**Recommended Books**

Purchase of a book is not necessary. However, background reading is strongly recommended.

Some of the material covered in first term lectures may be found in [1], and this also provides good background for second term lectures. Many other introductory statistics texts cover most of the basic techniques addressed. Note that various formulae and methods may differ slightly from book to book, and from lecture material to books.


**Calculators**

Approved electronic calculators are allowed in the examinations.
**Outline of course**

*Aim*: The module is designed to be a first statistics course. The emphasis is upon the understanding of real-life statistical problems, and develops the basic concepts and statistical methods by example.

**Term 1** (20 lectures)

**Sources of data**: Controlled experiments. Randomisation. Observational studies. Ethical practice.

**Descriptive statistics**: Displaying distributions: stem and leaf plots, histograms. Notation and summation formulae. Describing and summarising distributions: location (mean, median, percentiles); spread (variance, inter-quartile range); boxplots. Standardisation. Measurements and errors: outliers (link from boxplots), bias, randomness, chance errors, (informally) central tendency. Normal curve; areas under Normal curve; assessing Normality. Misleading graphs.


**Methods for more than two variables**: Least squares and multiple regression; two way tables, mean polish and median polish.

**Data analysis topics**: Chosen from the following. Non-linear least squares, smoothing, transformations, design of experiments.

**Terms 2 & 3** (21 lectures)


**Introducing inference**: Binomial distribution. Random sampling; the sample mean. Distribution of the sample mean. Central limit theorem. Normal approximation to binomial.

**Introduction to confidence intervals and hypothesis testing**: Generating confidence intervals. Basic ideas about hypothesis testing, type I and type II errors. Significance tests. P values. Sensible statistical reporting.

**Inferences for means of Normally distributed populations**: Procedures where the variance is known. Procedures where the sample size is large. t tests. Matched pairs problems. Comparing two population means. Comparing population variances. Comparing several population means (Analysis of variance).

**Methods for categorical data**: Fitting hypothesized frequencies to data. Fitting hypothesized probability distributions to data. Chi-square tests of homogeneity. Chi-square tests of independence.

**Distribution-free methods**: Spearman’s rank correlation coefficient. Mann-Whitney-Wilcoxon test, exact and approximate. Wilcoxon signed rank test, exact and approximate.
1.7.12 Brush Up Your Skills (1H Support Classes)  

Because of widening access, a broadening A-level syllabus and differences in the syllabuses of different boards, we facilitate revision and consolidation of the key skills required to embark on a mathematics degree through the “Brush Up Your Skills” course. The course covers material that most students will have seen at A-level, but as well as revision, the course is intended to cover any gaps there may be in any particular combination of A-level modules.

The course consists of 2 problems classes per week which complement the level 1 core modules. Attendance is not compulsory but is initially advised on the basis of a diagnostic test administered to all students at the beginning of the first term. The course is voluntary and does not form part of the degree, so students may attend only those sessions that deal with subjects where they feel weak. This facility is intended to help students take control of their own learning, recognize areas of weakness and use the resources available to improve them. It is the first step on the road to becoming an independent learner.

Recommended Books

Salas, S., Hille, E., Etgen, G., Calculus: One and Several Variables, J. Wiley & Sons, 10th ed., 2007
The Brush Up Your Skills course covers basic pre-calculus topics and broadly follows the Linear Algebra I and Calculus & Probability I syllabuses; most classes are led by questions posed by the students or suggested by the 1H lecturers so topics in other 1H courses (e.g. Analysis I and Problem Solving & Dynamics I) are also addressed.

**Basics:** number systems, basic manipulation, quadratic equations, polynomials, partial fractions, linear and non-linear inequalities, exponents and logarithms, topics in discrete mathematics.

**Functions:** definition, domain and range, graphs, linear and quadratic functions, composition, inverse, modulus function, hyperbolic functions.

**Coordinate Geometry:** equations and properties of straight lines, general equation of circle, centre and radius, Cartesian and parametric equations of curves.

**Trigonometry:** trigonometric functions and identities, inverse trigonometric functions, solution of trigonometric equations.

**Differentiation:** definition and properties, interpretation as slope, chain rule, sum, product and quotient rules, simple functions defined implicitly or parametrically, maxima and minima, Taylor and Fourier series, differential equations.

**Integration:** basic definition, as inverse of differentiation, as area under curve, integration methods, definite integrals, multiple integration.

**Vectors:** definition, basic properties and operations, magnitude, dot and cross products, vectorial geometry.

**Matrices:** definition, basic properties and operations, inverse, determinants.

**Probability:** permutations and combinations, set theory, Venn diagrams, calculus of probabilities, random variables, discrete and continuous distributions, moments, inequalities, approximations, law of large numbers.