3.5.2 CORE A CALCULUS – MATH1012 (29 lectures)

Dr A. Taormina

Calculus is elementary mathematics (algebra, geometry, trigonometry) enhanced by the limit process. Its invention is credited to Isaac Newton and Gottfried Leibnitz in the late seventeenth century. Leibnitz started his work in 1673, eight years after Newton, but initiated the basic modern notation for derivative and integration, dx and $\int$. From 1690 onward, calculus grew rapidly and reached its present state in roughly a hundred years.
This course will seek to consolidate and expand the knowledge you already have of this extremely important area of mathematics. It is designed to be completely accessible to the beginning calculus student without sacrificing appropriate mathematical rigour. The underlying emphasis is on the three basic concepts of calculus: limit, derivative and integral. Applications from the sciences, engineering, business and economics are often used to motivate or illustrate mathematical ideas. This course will be concerned with the nuts and bolts of calculus, while the Core B1 module will revisit the above concepts and provide a deeper knowledge.
Differential equations are introduced in connection with applications to exponential growth and decay. Many standard ordinary differential equations (ODE's) that appear frequently in applications are first and second order linear differential equations and are solved by methods that take advantage of their natural association with the technique of integration.
The last but equally important topic of the module is a first encounter with partial differential equations (PDE's), the most prominent of which are equations that relate two or more of the partial derivatives of an unknown function of several variables. The subject will be revisited in depth in the level 2 module 'Analysis in Many Variables II'. Here we concentrate on the technique of separation of variables and look in detail at one of the most famous PDE's: the heat diffusion equation.
The course will provide numerous exercises, some of them involving the use of the computer algebra package MAPLE.

Recommended Books
*Salas, Hille and Etgen, **One and several variables calculus**, 9th edition, Wiley, 2003 (hardback), ISBN 0-471-23120-7; £36.95.
M.L. Boas, **Mathematical Methods in the Physical Sciences**, Wiley, 1983 (hardback), ISBN 0-471-04409-1 (paperback is only available second hand: ISBN 0-471-09960-0).

The above books are strongly recommended. Both are useful in several modules at level 1 and 2 (Core A Geometry, Core B1, Core B2, Mathematical Physics II, Analysis in Many Variables). All mathematicians have to understand calculus, so there are many large books aimed at this vast market. A typical example of the genre is listed below. The more concise book by Haggarty might appeal to some students. All books are stocked in libraries in the University.

G.B. Thomas, **Calculus and analytic geometry**, Educational Technology, U.S.(2001) ISBN 0-877-78640-2; £49.99.
R. Haggarty, **Fundamentals of Mathematical Analysis**, Addison Wesley (2nd edition) ISBN 0-201-63197-0; £33.

Preliminary Reading: Revise A-level Core Mathematics material in your favourite books.
Calculators: The use of electronic calculators is forbidden in the examinations.

Outline of course **Core A Calculus MATH1012**

Aim: To master a variety of methods for solving problems and acquire some skill in writing and explaining mathematical arguments.

Term 1 (25 lectures)

Elementary Functions of a Real Variable: Domain and range. Graphs of elementary functions. Even and odd functions. Trigonometric and hyperbolic functions. Algebraic combinations and composition. One-to-one functions and inverses.

Limits and Continuity: Informal treatment of limits. Statement of main properties (uniqueness, calculus of limits theorem). Continuity at a point and on intervals.The pinching theorem and trigonometric limits. Statement of Intermediate Value Theorem and applications.

Differentiation : Derivative as slope of tangent line (the latter being a limit of chords). Differentiability and continuity. Differentiation formulas. Chain rule. Tangent as a linear approximation. Derivative as rate of change. Mean value and Rolle's theorems. Increasing and decreasing functions. Max-min problems. L'Hopital's rule. Partial differentiation.

Integration: Very basic treatment of definite integral in terms of Riemann sums. Indefinite integrals. Relation between the two types of integral (Fundamental theorem of Calculus). Use of partial fractions to integrate rational functions.

Ordinary Differential Equations: First order: separable, exact, homogeneous, linear. Second and higher order: linear with constant coefficients, importance of boundary conditions, reduction to a set of first order equations, treatment of homogeneous and inhomogeneous equations, particular integral and complementary function. Applications to particle dynamics (constant force, harmonic oscillator with damping).

Partial Differential Equations: Orthogonal functions and Fourier series. Convergence, periodic extension, sine and cosine series, half-range expansion. Separable PDE's. Solution of heat equation on a finite interval by separation of variables, solution of initial value problem using Fourier series.

Term 2 (4 lectures)

Taylor's Theorem: Statement of Taylor's theorem with integral and Lagrange remainders. Statement of Taylor's theorem in more than one variable. Examples of how to estimate remainder terms. Taylor series expansions of $e^x, \sin x, \sinh x, \log(1+x)$.

3.5.4 CORE A PROBABILITY – MATH1012 (22 lectures)

Dr I. M. MacPhee

The applications of probability are diverse, occurring in industry, mathematics, science, technology, medicine, agriculture, social science and many other fields. In this course, the theory of probability is developed but always with applications in mind. Among the topics covered are: probability axioms, conditional probability, special distributions, random variables, expectations, generating functions, applications of probability, laws of large numbers, central limit theorems.

Recommended Books

Students might buy one of the following books:

M.H. DeGroot & M.J. Schervish, **Probability and Statistics**, Int'l Edn, Addison-Wesley, ISBN 0-321-204735; £45 (This excellent book is also the recommended text for 2H Statistics and covers both courses very well).
J.H. McColl, **Probability**, Edwin Arnold, ISBN 0-340-614269; £14.99. (Inexpensive with reasonable coverage of the course.)
Y.A. Rozanov, **Probability Theory: a Concise Course**, Dover Publications, ISBN 0-486-63544-9; £8.95. (Very inexpensive. Covers the course quite well. Concise, as the title suggests.)
S. Lipschutz, **Theory and Problems of Probability**, Schaum Outline Series, McGraw-Hill, ISBN 007-0843-775, £16.99. (Good for basic problem solving and may be helpful for students who experience difficulty with probability.)

Calculators

The use of electronic calculators is forbidden in the examinations.

Outline of course **Core A Probability MATH1012**

Introduction to probability, chance experiments, sample spaces, events, assigning probabilities. Probability axioms and interpretations.

Conditional probability, theorem of total probability, Bayes theorem, independent events. Applications of probability.

Discrete random quantities, probability distributions and distribution functions, binomial, Poisson, Poisson approximation to binomial.

Expectations, variance, covariance, Chebyshev's inequality, weak law of large numbers.

Joint, marginal and conditional distributions, expectations of expectations.

Continuous random quantities, probability density functions, normal distribution, normal approximation to binomial.

Moment-generating functions.

Central-limit theorems.